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Spatial spin distribution of a skyrmion in a two-dimensional electron gas at a small g-factor

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Abstract. The effect of a weak Zeeman spin splitting on the spatial distribution and the energy of an isolated skyrmion in a 2D electron gas under a strong magnetic field at filling factor $\nu = 1$ is studied within the framework of the Hartree–Fock approximation. Finite Zeeman splitting introduces two different characteristic lengths into the problem, corresponding to the tail and the core of the spin distribution. The non-linear differential equation for the macroscopic spin density associated with the skyrmion is solved in the limit of very small g-factor, for which the tail of the skyrmion is much longer than its core radius. In this limit the Coulomb repulsion energy of the skyrmion, which determines the core radius, does not affect the shape of the spin distribution.

A two-dimensional electron gas (2DEG) in a strong magnetic field at filling factor $\nu = 1$ is a uniquely suitable system for investigating the interplay between the spin and the orbital degrees of freedom under the conditions of the quantum Hall effect (QHE). The magnetic-field-induced resolution of the single-particle energy spectrum into a series of discrete, but highly degenerate, spin-split Landau levels provides the energy gaps necessary for the occurrence of the integer QHE. The gap is defined as the energy necessary to create a spin-flipped electron and a hole, widely separated from each other in real space. In the limit in which the effective Landé g-factor is equal to zero, the system under consideration is equivalent to an isotropic 2D ferromagnet, which can be described using a three-component order parameter in a 2D coordinate space, i.e., by a model known as the non-linear O(3) model [1]. O(3) symmetry is known to be associated with some non-trivial topological invariants, which can lead to spontaneous creation of unusual topological point defects. At a filling factor $\nu = 1$, they have been shown to be skyrmions [2–6]. Such an isolated charged skyrmion is characterized by its topological charge (winding number) Q, which is equal to its electric charge [5].

The presence of large topological point defects [6] in the spin distribution of a 2D electron gas under a strong magnetic field has been recently associated with the observation in an optically pumped NMR experiment [7] of a strikingly large drop of the Knight shift as a function of the filling factor ν on either side of $\nu = 1$.

Further experimental support for the existence of such topological defects was provided recently in tilted-field magneto-transport experiments [8], as well as by interband optical

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transmission measurements [9], in which an anomalously large effective spin of the charge excitations near filling factor $\nu = 1$, was observed. The variation of the spin activation gap at $\nu = 1$ with the g-factor around $g = 0$, which was investigated very recently in electrical transport measurements under hydrostatic pressure [10], has been found to be in qualitative agreement with the current theoretical spin-texture (skyrmion) model [3].

The understanding of these rapidly accumulating experimental data should be based on a detailed knowledge of the energy spectrum and spatial spin distribution of skyrmions. While the energy spectrum completely determines the thermodynamical properties, the knowledge of the spatial spin structure is necessary for analysing the NMR-type experiments, which are very sensitive to the local inhomogeneities of the magnetic moments, due to the microscopic local origin of the hyperfine interaction.

It is now well known [3, 5] that the large Coulomb gap required for creating a widely separated quasielectron–quasihole pair (a large spin exciton [11, 12]) is reduced by a factor of 2 if the entire uniform spin distribution around the e–h pair created is twisted to form a widely separated skyrmion–anti-skyrmion pair.

In the approximation in which fourth-order spatial derivatives are neglected in the Hamiltonian, the total spin of a skyrmion with $Q = 1$ in the absence of the Zeeman energy is indefinite. This follows from the fact that a macroscopic number of reversed spins around the excitation core tend to reduce the exchange energy, while in this approximation the total excitation energy is independent of the skyrmion size.

In realistic 2D electron systems, the Zeeman spin splitting, though much smaller than the cyclotron energy, has a finite value. The Coulomb interaction acts to increase the skyrmion size while the Zeeman splitting tends to squeeze it. The interplay between these two factors determines the final distribution of spins in a skyrmion, and its characteristic length scales.

These length scales should play a significant role in cases in which the interactions between skyrmions become important. Such situations (i.e. for $\nu \approx 1$) have been recently investigated numerically within the HF approximation [13], and according to this work the ground state of the entire 2D electron gas may become unstable with respect to the formation of a skyrme crystal.

In the present paper we study analytically the effect of a very weak Zeeman splitting on the spatial distribution and the energy of an isolated skyrmion in 2D electron systems at filling factor $\nu = 1$. We show that skyrmion consists of a core, whose size is defined by the interplay between the Zeeman and Coulomb energies, and an additional length scale, l_{sk}, which determines the tail of the spin distribution.

It is well known [1] that the non-linear O(3) model consists of a vector field, $\boldsymbol{n}(\boldsymbol{r})$, with a unit norm, which is proportional to the macroscopic spin density of the 2D electron gas. The corresponding equation for the vector field $\boldsymbol{n}(\boldsymbol{r})$ can be obtained by variation of the HF energy functional with respect to $\boldsymbol{n}$ under the constraint $|\boldsymbol{n}|^2 = 1$. The resulting equation is [5]

$$\Delta \boldsymbol{n} - \boldsymbol{n}(\boldsymbol{n} \cdot \Delta \boldsymbol{n}) = l_{sk}^{-2}[(\hat{\boldsymbol{z}} \cdot \boldsymbol{n})\boldsymbol{n} - \hat{\boldsymbol{z}}] \tag{1}$$

where $\hat{\boldsymbol{z}}$ is a unit vector along the magnetic field direction (the z-axis), and the characteristic length scale

$$l_{sk}^{-2} = 2\sqrt{\frac{2}{\pi}}\,|g|(\tilde{a}_B/l_H^3). \tag{2}$$

Here g is the effective g-factor, which is different from the free-electron g-factor due to the crystal field, $l_H = (c\hbar/eH)^{1/2}$ is the magnetic length, and $\tilde{a}_B \equiv \kappa\hbar^2/m_0e^2$ is the effective Bohr radius (note that m_0 is the free-electron mass, and κ the dielectric constant). In the following we use the symbol g to denote the modulus of the effective g-factor.

The left-hand side of equation (1) has a standard form for the non-linear O(3) model, and the right-hand side is connected to the additive Zeeman term

$$-(g\mu_B H/4\pi l_H^2)\int \mathrm{d}^2 r\ (\hat{\boldsymbol{z}}\cdot\boldsymbol{n}) \tag{3}$$

in the energy functional.

It is clear from equation (1) that l_{sk} is a characteristic length scale of a skyrmion in the presence of a finite Zeeman splitting. This length is inversely proportional to the square root of the g-factor and becomes infinitely large in the limit of vanishing Zeeman splitting.

Taking the vectorial product of equation (1) with $\boldsymbol{n}$ produces the equation

$$\boldsymbol{n}\times\Delta\boldsymbol{n}+\frac{1}{l_{sk}^2}\boldsymbol{n}\times\hat{\boldsymbol{z}}=0. \tag{4}$$

It is easy to show that the three scalar equations corresponding to equation (4) are not linearly independent. Introducing the new complex variable $n \equiv n_x + \mathrm{i}n_y$, the two corresponding independent equations take the form

$$n\,\Delta n_z - n_z\,\Delta n + n/l_{sk}^2 = 0. \tag{5}$$

We will seek solutions of this equation in the form

$$\begin{aligned} n(\boldsymbol{r}) &= n^{(m)}(r)\mathrm{e}^{\mathrm{i}m\varphi} \\ n_z(\boldsymbol{r}) &= n_z^{(m)}(r) \end{aligned} \tag{6}$$

with boundary conditions which guarantee that the local spin at the core of the excitation (the origin) is completely reversed, i.e., $n_z^{(m)}(0) = -1$, while at infinite distance the spin density is uniform and aligned with the external field, i.e., $n_z^{(m)}(\infty) = 1$. Note, however, that the boundary condition at the origin ignores fluctuations of the spin distribution on a microscopical length scale, which seem to be significant very close to $r = 0$ for non-zero g-factor (see [14]).

These boundary conditions ensure that the angular momentum quantum number m is equal to the winding number Q of the skyrmion:

$$Q=\frac{1}{4\pi}\int \mathrm{d}^2 r\left(\boldsymbol{n}\cdot\left[\frac{\partial\boldsymbol{n}}{\partial x}\times\frac{\partial\boldsymbol{n}}{\partial y}\right]\right)=m[n_z^{(m)}(\infty)-n_z^{(m)}(0)]/2. \tag{7}$$

Since $\boldsymbol{n}$ is a unit vector, $|n|^2 + n_z^2 = 1$, we introduce the angular variable $\phi(r)$ such that $n^{(m)}(r) = \sin\phi(r)$, $n_z^{(m)}(r) = \cos\phi(r)$, and find the differential equation for ϕ:

$$\frac{\mathrm{d}}{\mathrm{d}x}(x\ \mathrm{d}\phi/\mathrm{d}x) = x\sin\phi + m^2\sin 2\phi/2x \tag{8}$$

where the dimensionless variable x is given by $x \equiv r/l_{sk}$. The skyrmion boundary conditions now take the following form: $\phi(0) = \pi$ and $\phi(\infty) = 0$.

The most interesting case corresponds to $m = 1$, in which the total number of reversed spins around the excitation core is known to diverge logarithmically as the effective g-factor tends to zero. The first term on the RHS of equation (8) describes finite Zeeman splitting. When this term is neglected, equation (8) becomes autonomous in the variable $t = \ln x$, and the imposed boundary conditions lead to the following solution:

$$\phi = 2\cot^{-1}(x/x_0) \tag{9}$$

where x_0 is an arbitrary parameter, determining the length scale $R = x_0 l_{sk}$ which describes the size of the skyrmion core region.

The corresponding vector field has the form

$$\boldsymbol{n} = \hat{\boldsymbol{x}}\sqrt{1 - n_z^2(r)}\cos\varphi + \hat{\boldsymbol{y}}\sqrt{1 - n_z^2(r)}\sin\varphi + \hat{\boldsymbol{z}}n_z(r)$$

with φ being the angle in the x, y (coordinates) plane, and $n_z(r) = (x^2 - x_0^2)/(x^2 + x_0^2)$. For the solution in the form of equation (9), the first term on the RHS of equation (8) is given by

$$x\sin\phi = 2x^2x_0/(x^2 + x_0^2) \tag{10}$$

and the second term by

$$\sin 2\phi/2x = 2x_0(x^2 - x_0^2)/(x^2 + x_0^2)^2. \tag{11}$$

Let us restrict ourselves to the case in which $x_0 \ll 1$. A simple estimate shows that in the region where $x \ll 1$ (i.e. for r far away from the exponential tail of the skyrmion; see below), expression (11) is much larger than (10), except for a very narrow region around $x = x_0$. Thus, in that case we may neglect the first term on the RHS of equation (8) with respect to the second term, and the corresponding solution is (9) (see, however, [14]).

Next, we may consider the region $x_0 \ll x \ll 1$ (i.e. for r far outside the skyrmion core, but deep inside the region defined by l_{sk}), where $\phi \approx 2x_0/x \ll 1$. Throughout the entire region $x \gg x_0$ (where $\phi \ll 1$), we may restrict ourselves to an expansion of the RHS of equation (8) in small ϕ. To first order in ϕ we thus obtain

$$\frac{\mathrm{d}}{\mathrm{d}x}(x\ \mathrm{d}\phi/\mathrm{d}x) = (x + 1/x)\phi \tag{12}$$

which may be rewritten in the form

$$x^2\ \mathrm{d}^2\phi/\mathrm{d}x^2 + x\ \mathrm{d}\phi/\mathrm{d}x - (1 + x^2)\phi = 0. \tag{13}$$

The general solution of this equation is

$$\phi(x) = \alpha I_1(x) + \beta K_1(x) \tag{14}$$

where $I_1(x)$, $K_1(x)$ are modified Bessel functions and α and β are arbitrary constants. The boundary condition at $r = \infty$ determines the first constant to be $\alpha = 0$, while the asymptotic behaviour of ϕ as derived from equation (9) is $\phi \sim 2x_0/x$. On the other hand, for $x \ll 1$, equation (14) with $\alpha = 0$ implies that $\phi \approx \beta/x$. Comparing these expressions we determine that $\beta = 2x_0$.

The corresponding distribution of spin polarization for a skyrmion has the form

$$n_z(x) = \begin{cases} (x^2 - x_0^2)/(x^2 + x_0^2) & \text{for } x \ll 1 \\ 1 - 2x_0^2[K_1(x)]^2 & \text{for } x \gg x_0. \end{cases}$$

In the asymptotic region, $x \gg 1$,

$$n_z(r) \approx 1 - \frac{\pi R^2}{r l_{sk}}\exp(-2r/l_{sk}).$$

Using these expressions we can calculate now the total number S_z of reversed spins associated with a skyrmion:

$$S_z = \frac{1}{4\pi l_H^2}\int [1 - n_z(r)]\ \mathrm{d}^2r$$

which leads to the following expression:

$$S_z = \left(\frac{x_0 l_{sk}}{l_H}\right)^2 \ln(2/x_0\sqrt{\mathrm{e}}) \tag{15}$$

where e is the natural logarithm base.

The Zeeman energy associated with the reversed spins is

$$\Delta E_Z = g\mu_B H S_z = \frac{1}{4}\sqrt{\frac{\pi}{2}}\frac{e^2 x_0^2}{\kappa l_H}\ln(2/x_0\sqrt{\mathrm{e}}). \tag{16}$$

The other correction to the skyrmion energy is associated with the non-uniformity of the spin density appearing in the HF energy functional [5], i.e.,

$$\sqrt{\frac{\pi}{2}}\frac{e^2}{\kappa l_H}\frac{1}{32\pi}\int(\boldsymbol{\nabla}\cdot\boldsymbol{n})^2\ \mathrm{d}^2 r. \tag{17}$$

It is easy to show that

$$\int(\boldsymbol{\nabla}\cdot\boldsymbol{n})^2\ \mathrm{d}^2 r = 2\pi\int_0^\infty\left[\left(\frac{\mathrm{d}\phi}{\mathrm{d}x}\right)^2 + \left(\frac{\sin\phi}{x}\right)^2\right]x\ \mathrm{d}x = 8\pi(1 + x_0^2/2)$$

and thus the corresponding expression for the energy (17) is

$$\frac{1}{4}\sqrt{\frac{\pi}{2}}\frac{e^2}{\kappa l_H}(1 + x_0^2/2).$$

Thus, the total correction to the energy of the skyrmion due to the effect of a non-zero g-factor, which consists of both the Zeeman energy and the correction associated with the non-uniform spin density, equation (17), can be written as

$$\Delta E = \frac{x_0^2}{4}\sqrt{\frac{\pi}{2}}\frac{e^2}{\kappa l_H}\ln(2/x_0). \tag{18}$$

Note that $R = x_0 l_{sk}$ may be regarded as the radius of the skyrmion core region, which is a very small fraction of the total skyrmion extension, l_{sk}, provided that $x_0 \ll 1$.

At this point we conclude that the total number of reversed spins within a single skyrmion, as well as the total skyrmion energy (equations (15) and (18), respectively) increase quadratically (we neglect a weak logarithmic dependence) with the radius of the skyrmion core region. Under these circumstances it is impossible to avoid the collapse of the skyrmion core.

This is, of course, not a physical result: it just reflects the absence of the Coulomb self-energy repulsion associated with the skyrmion charge distribution in our analysis. This interaction is of higher order in the gradient expansion than the terms considered in our calculations so far. In the small-g-factor limit considered here, the direct Coulomb energy term is the dominant one.

To calculate the direct Coulomb energy we use the expression

$$E_C = \frac{1}{2\kappa}\int\delta n(r_1)\,\delta n(r_2)\ \mathrm{d}\varphi_1\ \mathrm{d}\varphi_2\ V(|\boldsymbol{r}_1 - \boldsymbol{r}_2|) \tag{19}$$

where

$$\delta n(r) = \frac{1}{4\pi}\frac{\mathrm{d}n_z}{\mathrm{d}r}\ \mathrm{d}r$$

and $n_z(r) \equiv (r^2 - R^2)/(r^2 + R^2)$. Equation (19) can be presented in the following form:

$$E_C = \frac{1}{4\pi\kappa R^2}\int_0^\infty x^3\widetilde{V}\left(\frac{x}{R}\right)K_1^2(x)\ \mathrm{d}x \tag{20}$$

where $K_1(x)$ is modified Bessel function, and $\widetilde{V}(x)$ is the Fourier component of the effective Coulomb potential $V(r)$. In the absence of screening, $\widetilde{V}(x) = 2\pi e^2/x$. The screening changes the form of the Fourier component $\widetilde{V}(x)$. The Fourier component of the effective

Coulomb potential $V(r) = e^2 \exp(-r/r_0)/r$ is equal to $\widetilde{V}(x) = 2\pi e^2/\sqrt{x^2 + 1/r_0^2}$. In the case in which the asymptotic behaviour of the effective Coulomb potential is of dipolar type, the Fourier component is $\widetilde{V}(x) = 2\pi e^2/(x + 1/r_0)$.

In the absence of screening, the Coulomb energy (20) is

$$E_C = \frac{3\pi^2 e^2}{2^6 \kappa R}. \tag{21}$$

It is easy to show that for the case in which the core radius R is much larger than the characteristic screening length r_0, the final expression for the Coulomb energy (20) does not depend on the exact form of $\widetilde{V}(x)$, and is

$$E_C = \frac{e^2 r_0}{3\kappa R^2}. \tag{22}$$

To find the core radius R, we have to minimize the total energy with respect to R. In the absence of screening, the total energy is

$$E_C + \Delta E = \frac{e^2}{2\kappa l_H}\left[\frac{3\pi^2}{2^5 R} + \frac{g\tilde{a}_B}{l_H^2}\left(\frac{R}{l_H}\right)^2 \ln\left(\frac{2l_{sk}}{R}\right)\right]. \tag{23}$$

The resulting transcendental equation for the core radius R can be approximately solved to give

$$\left(\frac{R}{l_H}\right)^3 = \frac{9\pi^2 l_H}{2^5 g\tilde{a}_B} \Big/ \ln\left(\frac{0.4 l_H}{g\tilde{a}_B}\right). \tag{24}$$

For the screened Coulomb potential, the core radius R is

$$\left(\frac{R}{l_H}\right)^4 = \frac{8r_0}{3g\tilde{a}_B} \Big/ \ln\left(\frac{3\pi l_H^2}{4\mathrm{e}^2 r_0 \tilde{a}_B g}\right). \tag{25}$$

Thus, neglecting the fourth-order correction to the exchange energy in the gradient expansion of the energy functional, our final result for the energy of a widely separated skyrmion–anti-skyrmion pair in the absence of screening can be written in the form

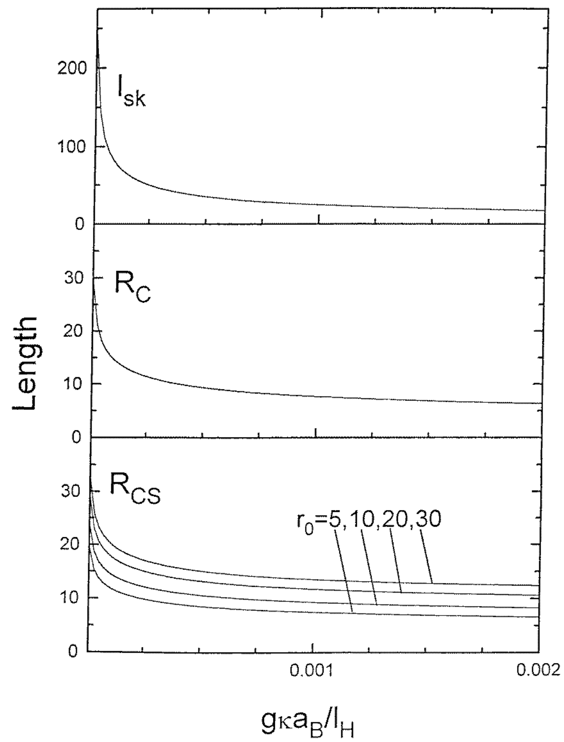
$$E(g) = \frac{1}{2}\sqrt{\frac{\pi}{2}}\frac{e^2}{\kappa l_H}\left[1 + \frac{3\pi}{8}\left(\frac{18}{\pi}\right)^{1/6}(\tilde{g}|\ln(2.5\tilde{g})|)^{1/3}\right] \tag{26}$$

where $\tilde{g} \equiv g(\tilde{a}_B/l_H)$ determines the ratio of the Zeeman splitting to the Coulomb energy.

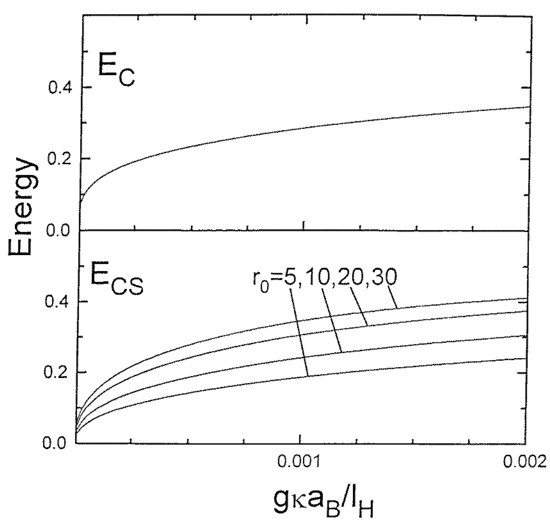
The corresponding expression for the screened Coulomb potential is

$$E(g) = \frac{1}{2}\sqrt{\frac{\pi}{2}}\frac{e^2}{\kappa l_H}\left[1 + 4\left(\frac{r_0\tilde{g}}{3\pi l_H}\ln\left(\frac{3\pi l_H}{4r_0\tilde{g}\mathrm{e}^2}\right)\right)^{1/2}\right]. \tag{27}$$

In conclusion, we have considered the problem of skyrmions in 2D electron systems under strong magnetic fields at filling factor $\nu = 1$ to determine the effect of a weak Zeeman splitting on the spin distribution of an isolated charged skyrmion. We have found that the Zeeman splitting introduces a new length scale l_{sk} which determines the tail of the spin distribution. On including the Coulomb self-energy repulsion of the skyrmion charge distribution, a second characteristic length scale, R, for the skyrmion spin distribution is determined by the finite g-factor. This length scale corresponds to the skyrmion core region. The results for the core radius of the skyrmion (24) and the corresponding expression for the energy of the skyrmion–anti-skyrmion pair in the absence of screening (26) are similar to ones obtained by Sondhi *et al* [3].



(a)



(b)

Figure 1. Characteristic lengths (a) (in units of the magnetic length, l_H), and corrections to the energies due to the effect of a non-zero g-factor (b) (in units of $\frac{1}{2}\sqrt{\pi/2}e^2/\kappa l_H$), as functions of the normalized effective g-factor $\tilde{g} = g\kappa a_B/l_H$. l_{sk} is the skyrmion extension; R_C and E_C are, respectively, the radius of the skyrmion's core and the energy of the skyrmion–anti-skyrmion pair for a bare Coulomb potential. R_{SC} and E_{SC} are the corresponding radii and energies in the case of a screened Coulomb potential with several screening lengths r_0, marked in the figure in units of the magnetic length.

The explicit expression for the skyrmion spin density, obtained from equation (14), is a new result, which is readily derived within our analytical approach. The derivation is restricted to the limit of very small g-factor, i.e., when l_{sk} is much larger than the core radius, R. In this limit, the shape of the spin distribution within the core of the skyrmion is not affected, neither by the Zeeman splitting nor by the Coulomb energy, and is the same as that of an ideal skyrmion.

It should be stressed that the determination of the core radius, R, described in technical terms above, has a clear physical meaning. It is the result of a competition between the Zeeman energy (16), which grows quadratically with R, and so forces the skyrmion to shrink, and the Coulomb self-energy repulsion of the skyrmion, which decreases with R, and so forces it to inflate.

One should note, however, that although the Zeeman energy grows quadratically with R, it is multiplied by a coefficient proportional to a very small g-factor. The Coulomb energy (in the absence of screening) decreases like $1/R$, but with a coefficient which is much larger than that of the Zeeman energy term. Therefore the Coulomb energy can be compared with Zeeman energy only at very large R.

The resulting radius R, equation (24), grows weakly to infinity as the g-factor goes to zero, thus reflecting the importance of the long-range Coulomb repulsion associated with the skyrmion charge in the zero-g-factor limit. It should be noted that in the presence of screening the Coulomb energy decreases more rapidly with R, which results in a smaller value of R at the same g-factor. Accordingly, the correction to the energy of the skyrmion–anti-skyrmion pair is smaller in the case of a screened Coulomb potential than in the case of the unscreened potential.

Our findings are illustrated in figure 1, where realistic values of the relevant parameters are used. In particular, the values of the normalized effective g-factor, $\tilde{g}$, considered (i.e. $|\tilde{g}| \leqslant 0.002$) are in the range studied experimentally by Maude *et al* [10], where the skyrmion size was estimated to approach $33 l_H$ in the $g \to 0$ limit. It is clearly seen that the effect of screening does not qualitatively change the g-dependence obtained for the bare Coulomb interaction, which is significantly sharper than the experimentally measured behaviour near $g = 0$.

The smearing of the sharp behaviour near $g = 0$ is most probably due to the effect of long-range potential fluctuations in the heterostructure, neglected in our model, which provide widely separated trapping sites for skyrmion–anti-skyrmion pairs with energies well below their intrinsic Coulomb gap [15]. This effect seems also to explain the large difference between the experimentally observed gap and the theoretically predicted one.

Acknowledgments

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It is shown there that at x much smaller than x_0 the first term on the RHS of equation (8) can drastically affect the solution of this equation. The function $\phi(x)$ very close to the origin has a rather complicated behaviour, which is not reflected in the estimates (10), (11): with decreasing x, $\phi(x)$ first closely follows the ideal skyrmion behaviour, equation (9), nearly reaching the value π, but then changes in a rather complicated way. Since we are interested here only in the behaviour of the spin distribution over length scales comparable to or larger than the skyrmion core radius R (which is much larger than the magnetic length in our small-g-factor limit), this singular behaviour can be ignored.
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